Performance Based Learning and Assessment Task

Throwing a Football

I. ASSESSMENT TASK OVERVIEW & PURPOSE:

Students will be using a variety of algebraic concepts and skills dealing with quadratic functions. In this task, students will help the high school football quarterback with timing his passes. Students will have to make connections between the football's parabolic path and use mathematics to help guide the quarterback. Students will work in groups of 2 to gather data, explore different options for completing the task, and demonstrate their ability to answer follow-up questions dealing with the ball's position (height) with respect to time. Students may choose to use research, reasoning, and mathematics to come up with solutions that will be presented to the class. This activity helps students develop and implement a plan of action, determine what information is needed to help them make the best decisions and find solutions, and make connections between quadratic equations and a football's trajectory.

II. UNIT AUTHOR:

Yolonda Shields, Staunton River High School, Bedford County Public Schools

III. COURSE:

Algebra I / Algebra II

IV. CONTENT STRAND:

Functions

V. OBJECTIVES:

- The student will solve quadratic equations in two variables, including b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets; c) solving quadratic equations algebraically and graphically; f) solving real-world problems involving equations. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.
- The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.
- The student will investigate and analyze functions algebraically and graphically. Key concepts include a) domain and range, including limited and discontinuous domains and ranges; b) zeros; c) x- and y-intercepts.

VI. REFERENCE/RESOURCE MATERIALS:

TI-83 Plus (or higher) Graphing Calculator, computer, pencil, paper, graph paper, stopwatch, footballs, rulers, Internet, GeoGebra, Geometer's Sketchpad, or Desmos Software (optional),

Assessment Rubric, copy of Performance Task, word processing software (i.e Microsoft Word or Google Docs), copy of Benchmarks.

VII. PRIMARY ASSESSMENT STRATEGIES:

While students are working on the Performance Based Assessment task in groups of 2, the teacher will be evaluating the students to make sure each stay on task. If the teacher notices students are off task, then students will be referred to the attached rubric/list, which both the student and teacher can use as a reference, a checklist, and a rubric. The assessment list for each of the activities will contain all the essential components for this mathematics activity. This includes the mathematics content, process skills, and requirements for the finished product.

VIII. EVALUATION CRITERIA:

Students will be evaluated on their completion of the activity and by their final product. The attached Assessment List and Rubric provides more details on what is expected. Benchmarks are also attached to end of this assessment task.

IX. INSTRUCTIONAL TIME:

Depending on the students' prior experience and comfort level with problem solving, reasoning, and mastery of the mathematics needed to find the solution, it could take up to an entire class period (90 minutes) on a block schedule to come up with a plan, implement the plan, and do any necessary research for addressing and answering the authentic problems.

Throwing a Football /Task 1

Strand
Functions

Mathematical Objective(s)

- The students will use their knowledge of solving quadratic equations in two variables, both algebraically and graphically, to solve this real-world problem involving equations.
- The students will successfully use any available tools or technologies to assist them, such as a graphing calculator.
- The students will be able to investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.
- The students will investigate and analyze functions algebraically and graphically and be able to recognize how the key concepts of domain and range and their restrictions apply to this real world situation.

The mathematical goal of this activity is for students to use their current knowledge and to seek out additional information needed to solve this authentic football problem. The goal is also to help students use mathematics, collaboration, logical thinking, and research to gain additional insight and familiarity of quadratic equations and their relationships to a variety of real world examples. Students will have a deeper appreciation of mathematics and its valuable role in many areas, including football. Students may also use word processing software to organize their ideas and answer questions that will be a part of their presentation (bonus points). Nevertheless, students will gain valuable skills as they gather information and communicate their mathematical ideas.

Related SOLs

- **SOL A.4** (Solving quadratic equations in two variables, both algebraically and graphically; solving real-world problems involving equations)
- **SOL AII.8** (Investigating and describing the relationships among the solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expressions)
- **SOL AII.7** (Investigating and analyzing functions algebraically and graphically)

NCTM Standards
Students will engage in problem solving, communicating, reasoning, connecting, and representing as they…

- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Algebra:

Students will:

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships.
- Draw reasonable conclusions about a situation being modeled

Measurement:

Students will:

- Make decisions about units and scales that are appropriate for problem situations involving measurement
- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements.
- Use unit analysis to check measurement computations

Problem Solving:

Students will:

- Build new mathematical knowledge through problem solving;
- Solve problems that arise in mathematics and in other contexts;
- Apply and adapt a variety of appropriate strategies to solve problems;
- Monitor and reflect on the process of mathematical problem solving.

Connections:

Students will:

- Organize their mathematical thinking through discussion with peers
- Communicate their thinking clearly to teacher and peers
- Analyze and evaluate the mathematical thinking and strategies of their partners
- Use the language of mathematics to express mathematical ideas precisely
- Recognize and apply mathematics in contexts outside of mathematics
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole

Representation:

Students will:

- Create and use representations to record and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations.
- Use representations to model and interpret physical and mathematical phenomena.

Materials/Resources

TI-83 Plus (or higher) Graphing Calculator, computer, pencil, paper, graph paper, stopwatch, footballs, rulers, Internet, GeoGebra, Geometer's Sketchpad, or Desmos Software (optional),

Assessment Rubric, copy of Performance Task, word processing software (i.e Microsoft Word or Google Docs), copy of Benchmarks.

Assumption of Prior Knowledge
- Knowledge of quadratic equations and how to solve them
- Ability to recognize parabolas, even though they are oriented differently when it comes to the path of a thrown football
- Familiarity with the variety of ways of solving Quadratic Equations (i.e. graphing, factoring, Quadratic Formula)
- Experience with finding and graphing data
- Understanding situations in which only positive roots should be used
- Success with proportional reasoning and doing distance conversions (i.e. meters to yards)
- Some experience with graphing calculator technology and Geometer's Sketchpad / GeoGebra Software; Experience with using Word Processing Software

Introduction: Setting Up the Mathematical Task
- Discuss the activities for the day (refer to displayed Agenda)
- Motivating activity to introduce the goal of the task/activity (Experiment: Using a stopwatch, determine the time it takes for you to throw a football a certain distance. If the distance remained the same each throw, what would cause the time to increase or decrease?
- Student discussion about why these differences occurred. Students would be able to see that if the distance remained the same, then the velocity at which the football was thrown would have an effect on the time it was in the air. This would lead to the mathematical performance task.
- Distribute the task and assessment rubrics to the students; teacher will give an outline of the performance assessment task and the timeframe for completion as students look at the typed version that is passed out to them.
- Students can pick a partner or the teacher can create the groups
- In their groups of 2, students will begin brainstorming steps for addressing this real-world task
- Teacher will act as facilitator and will ask questions or give prompts to the students, such as things they should consider as a way to help guide their understanding. The teacher will reinforce the idea that he or she is going to be in this role and that the students will be responsible for developing a plan and implementing that plan to come up with a solution.
- Students may find the need to do some research or brainstorm how the path of a football could resemble a mathematical concept.
- Students will be asked to draw upon their prior knowledge to come up with solutions

- Teacher will help students understand the task by effectively answering their questions if they arise.
- Students will have to use mathematics to solve the problem, but will be given access to a computer and the Internet to help them.
- To make the students' mathematical thinking and understanding public, students will collaborate with their partners, come up with a solution or solutions, and share their ideas through a culminating activity that involves the presentation of their information.

Student Exploration

- Students will be collaborating with their partner to come up with their solution. The students will be actively using mathematics, research, and exploration to do a variety of calculations. Students will have access to the computer for research, typing their results, and using GeoGebra/Geometer's Sketchpad Software if desired. They will also have access to the Internet and a graphing calculator to help them with these calculations.
- Students will analyze and describe the path the football takes after being released from the quarterback's hands.
- Students will be tasked with making a connection between their observations and the mathematical concept of quadratic functions.
- There are many different variables to consider, such as time, velocity, height, etc., but this activity focuses on two of these variables.
- Possible misconceptions or errors could involve students not recognizing that even though there are two zeros to this quadratic function, only one of them will be relevant because time will never be negative. In this situation, we are only concerned about its positive root.
- Students can experiment with reconstructing the image using GeoGebra or Geometer's Sketchpad software.
- Higher-order questioning will be used, such as: Are there some values that are irrelevant? If so, discuss and explain why.
- The teacher also has the option of making this task even more challenging by having students solve related problems requiring them to work backwards.

Monitoring Student Responses
- I expect students to work together as they communicate their thinking and their new knowledge with each other and with the teacher.
- The teacher will help students with any clarification needs and assist students who are having difficulties by helping them connect their previous experiences to these new ones.
- If there are students who are ready to move forward, differentiation will be used and the students can explore additional ways to solve the problem, additional topics (listed earlier

in the student exploration section, etc.). This information could also be included in the final product.

- Students who are ready to prepare their class presentation can go ahead and move to the computer to begin typing out key ideas/information (they will earn bonus points for taking this additional step to organize their work and enhancing its neatness).

Closure will involve the students sharing their results through a culminating activity—a class presentation. This will be followed by a group discussion and with the teacher providing feedback.

Performance Based Assessment Task

Name(s): _____

_____

Date: _____

Algebra Teacher: _____

Throwing a Football

The quarterback of your high school football team is struggling with timing his throws to his receivers. Sometimes he overthrows and at other times he does not throw far enough. As a result, the ball gets intercepted often. The quarterback comes to you and asks for your mathematical advice and you tell him that you will get back with him within a few days because you need time to gather and organize some additional mathematical information that you feel will be useful to him. While brainstorming, you realize that it would be very beneficial to get the assistance of another student with whom you could exchange ideas with and devise a plan for addressing this real-world problem.

Here is your task:
1. Brainstorm and make a list of all the variables that can be considered / any useful information that is available and/or needs to become available in order for your group to successfully help the quarterback. Each group member should contribute at least 3 suggestions. Make sure you sign your initials beside your suggestions.
2. Using your list created in step #1, record any information you found if any.
3. Come up with a mathematical representation of the aforementioned information.
4. Consider the position of the ball at time zero and the following question: When will the ball hit the ground if no one is there to catch the ball?
5. Determine how much time would have to pass before the receiver could catch the ball at 1.1 meters.
6. Graph a model showing the path the football takes after being released from the hands of the quarterback.
7. When is the ball at its highest point? How do you know? Explain and show how mathematics can be used to prove this.
8. What is the highest the ball can go? Explain how you found this value.
9. If the receiver runs 30 yards in 5 seconds, when should the quarterback release the ball?
10. Give some additional examples (at least 2) of other objects that could follow a parabolic trajectory.

<u>Class Presentation</u>

You will present and share you and your partner's solutions for the quarterback to the math class in any format you choose (letter, conversation, etc.) If you type the key concepts / information you used for your 2-3 minute presentation, you will earn bonus points! Be prepared for your teacher and your classmates to ask questions and/or give you feedback after the presentation. You want to make sure you create a quality and neat product that you can share with the quarterback.

*REFER TO THE ATTACHED ASSESSMENT LISTS & RUBRICS FOR A CHECKLIST/INFORMATION ON HOW YOU & YOUR PARTNER WILL BE GRADED!

Assessment List, Rubric, and Benchmarks

#	Element	Point Value	Earned Assessment Self	Teacher
1	Student contributed at least three suggestions in the initial planning discussions with his/her partner (indicated by initials).	2		
2	Student developed a plan for coming up with solutions for the task.	2		
3	Student documented any research done and the information gathered to address this real-world problem.	2		
4	Student recorded measurement findings while completing the activity and used proper mathematics to verify solutions.	2		
5	Student correctly identified the path that the football takes while in the air.	2		
6	Student explained their mathematical reasoning behind their model selection	2		
7	Student correctly created a quadratic equation that models the situation	2		
8	Student can show why their equation works and correctly substitutes values in for the variables.	2		
9	Student identifies the initial height of the football and describes how it was found.	2		
10	Student correctly estimates the time it would take for the ball to hit the ground if no receiver catches the ball by using the equation	2		
11	Student correctly estimates the time it would take for the receiver to catch the ball by taking the catching height into consideration along with the equation	2		
12	Student graphs an accurate model representing the football's trajectory in the air.	2		
13	Student uses mathematics to determine when the ball reaches its maximum height.	2		

#	Element	Point Value	Self	Teacher
14	Student determines the maximum height the football travels and explains how this value was found	2		
15	Given the speed at which the receiver runs, the student accurately determines the most ideal time for the quarterback to release the football.	2		
16	Student gives at least 2 additional objects that would also follow the football's parabolic trajectory.	2		
17	Student answers all of the questions and provides reasoning for each answer	2		
18	Student actively participates in a 2-3 minute class presentation	2		
19	Student answered all of the higher-order questions posed to him or her by the teacher and the classmates after the presentation	2		
20	Student's work and presentation is well-organized	2		
21	Student's work is neat	2		
22	Student can explain their reasoning for their ideas, formulas, and work shown from activity.	2		
23	Student uses Word Processing Software to type presentation key ideas / information (**Bonus Points!**)	2		
	Total (Out of 44)			

	RUBRIC FOR ACTIVITY 1			
#	Element	0	1	2
1	Student contributed at least three suggestions in the initial planning discussions with his/her partner (indicated by initials).	Student did not make any contributions	Student contributed 1 or 2 times	Student contributed 3 or more times
2	Student developed a plan for coming up with solutions for the task.	No evidence of a plan provided	Evidence of plans, but incomplete	Evidence of complete plans
3	Student documented any research done and the information gathered to address this real-world problem	No documentation	Minimal documentation	Sufficient documentation
4	Student recorded measurement findings while completing the activity and used proper mathematics to verify solutions.	Measurements are not provided	Some measurements are provided.	All measurements are provided
5	Student correctly identified the path that the football takes while in the air.	Student does not correctly identify the path	Student attempts to identify the name or the attributes of the football's path, but does so incorrectly.	Student correctly identifies the path of the football
6	Student explained their mathematical reasoning behind their model selection	Does not explain	Attempts to explain, but does not do so correctly	Clearly explains their reasoning
7	Student correctly created a quadratic equation that models the situation	No equation developed	Equation developed, but not correct	Correct and logical equation developed

#	Element	0	1	2
8	Student can show why their equation works and correctly substitutes values in for the variables.	No/unclear demonstration about why the equation works; No indication of appropriate use of substitution into these formulas	Only one of the 2 requirements are met and/or correct. For example, some values are substituted correctly into these formulas	Both of the requirements are met and correct. All values are substituted correctly into these formulas
9	Student identifies the initial height of the football and describes how it was found.	Initial height and description not given	Initial height given, but no description	Both the initial height was given along with the description
10	Student correctly estimates the time it would take for the ball to hit the ground if no receiver catches the ball by using the equation	Student does not estimate the time using the equation	Student attempts to estimate the time, but does not use correctly use the equation.	Student correctly estimates the time using the equation
11	Student correctly estimates the time it would take for the receiver to catch the ball by taking the catching height into consideration along with the equation	Student does not estimate the time using the equation	Student attempts to estimate the time, but does not use correctly use the equation or makes a minor mathematical error.	Student correctly estimates the time using the equation
12	Student graphs an accurate model representing the football's trajectory in the air.	Graph of the football's trajectory is not present	Graph of the football's trajectory is present, but inaccurate	Graph of the football's trajectory is present and accurate.
13	Student uses mathematics to determine when the ball reaches its maximum height.	Does not use math to find the time the ball reaches its maximum height.	Student finds a time for when the ball reaches maximum height, but it is inaccurate.	Student correctly uses mathematics to determine when the football reaches its maximum height.

#	Element	0	1	2
14	Student determines the maximum height the football travels and explains how this value was found	No maximum height or explanation given	Student finds a maximum height for the football, but it and the explanation are inaccurate.	Student correctly determines the football's maximum height and gives a clear explanation as to how this value was found.
15	Given the speed at which the receiver runs, the student accurately determines the most ideal time for the quarterback to release the football.	Student does not determine the most ideal time for football release.	Student determines an ideal time for football release, but it is inaccurate.	Student correctly determines the ideal time for the football to be released.
16	Student gives at least 2 additional objects that would also follow the football's parabolic trajectory.	Student does not name any additional objects.	Student only names 1 additional object	Student names 2 or more additional objects
17	Student answers all of the questions and provides reasoning for each answer	None of the questions were answered and no reasoning was given	Some of the questions were answered along with the reasoning	All of the questions were answered and the reasoning for each were provided
18	Student actively participates in a 2-3 minute class presentation	Student does not participate	Student passively participates	Student actively participates
19	Student answered all of the higher-order questions posed to him or her by the teacher and the classmates after the presentation	No follow-up questions answered	Follow-up questions answered, but are not correct or clear	Follow-up questions answered logically and correctly (or close to correct)
20	Student's work and presentation is well-organized	No evidence of organization	Not fully organized	Well-organized work and presentation
21	Student's work is neat.	Lacks neatness	Needs improvement	Neat and legible
22	Student can explain their reasoning for their ideas, formulas, and work shown from activity.	Student provides no explanation	Explanation provided with logical flow, but is mostly incorrect	Explanation provided with logical flow and is mostly correct
23	Student uses Word Processing Software to type presentation key ideas / information (**Bonus Points!**)	Did not use this software	Did not use this software	Used this software

Benchmark

1) List of variables to be considered: time, distance, air resistance, height that the ball travels, velocity, acceleration due to gravity, initial height of the football, the final height of the football (Student #1: initials beside the first 5; Student #2 initials beside the last 4 suggestions – The following information was a collaborative brainstorming effort:))

a) The task tells us to ignore air resistance, so we do not need to research that topic. We can use the Internet or our Science textbook to find the acceleration due to gravity on Earth. (We researched this on the internet and found that this value is 9.81 m/s^2.

b) We need to find out more about the other variables. We could possibly gather our own measurements to help us model this authentic situation. Maybe we should actually gather data using ourselves as a trial run and then we can go to a football practice and gather data from the quarterback and one of his receivers. Then, we would be able to give him valuable and useful information.

2) We were not too successful throwing the football ourselves, so we went to football practice the next day. We decided to measure the distance from the ground to the point of where the quarterback typically releases the ball to find the initial height of the ball at time zero. We found the height to be 1.7 meters. We then ask one of the receivers to let us measure the distance from the ground to the point where he typically catches the ball, which is at 1.1 meters. We even got the coach involved! The coach let us know that the typical velocity of the quarterback's passes was 15 m/s. We used this valuable information and our mathematical knowledge to determine the following equation to be used for the height of the football (Note: We ignored air resistance in all our responses): Let h represent height and t represent time

$$h = 1.7 + 15t - ½ (9.81) t^2$$

This information aided in our feedback to the quarterback.

3) The model looks like a parabola. Here is a picture we found by doing a Google Search:
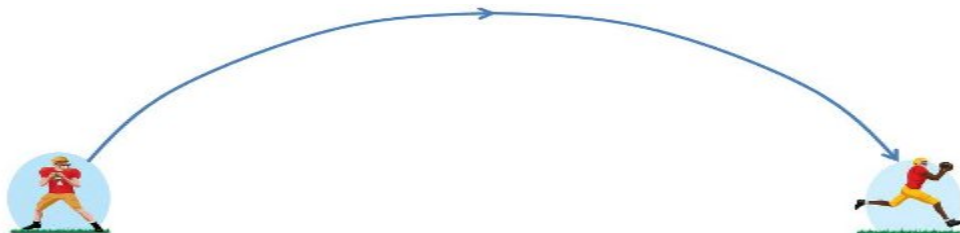
4) If the ball is 1.7 meters above the ground at time zero, the ball will hit the ground after 3.168 seconds. Here is our work:

h = 1.7 + 15t − ½ (9.81) t^2

0 = 1.7 + 15t − 4.905 t^2

We decided to graph this quadratic equation (y = 1.7 + 15x − 4.905x^2) in order to find the x-intercept (place where the ball hits the ground). The x-value gives us the amount of time that has passed. We used Desmos Software. Here is the graph:
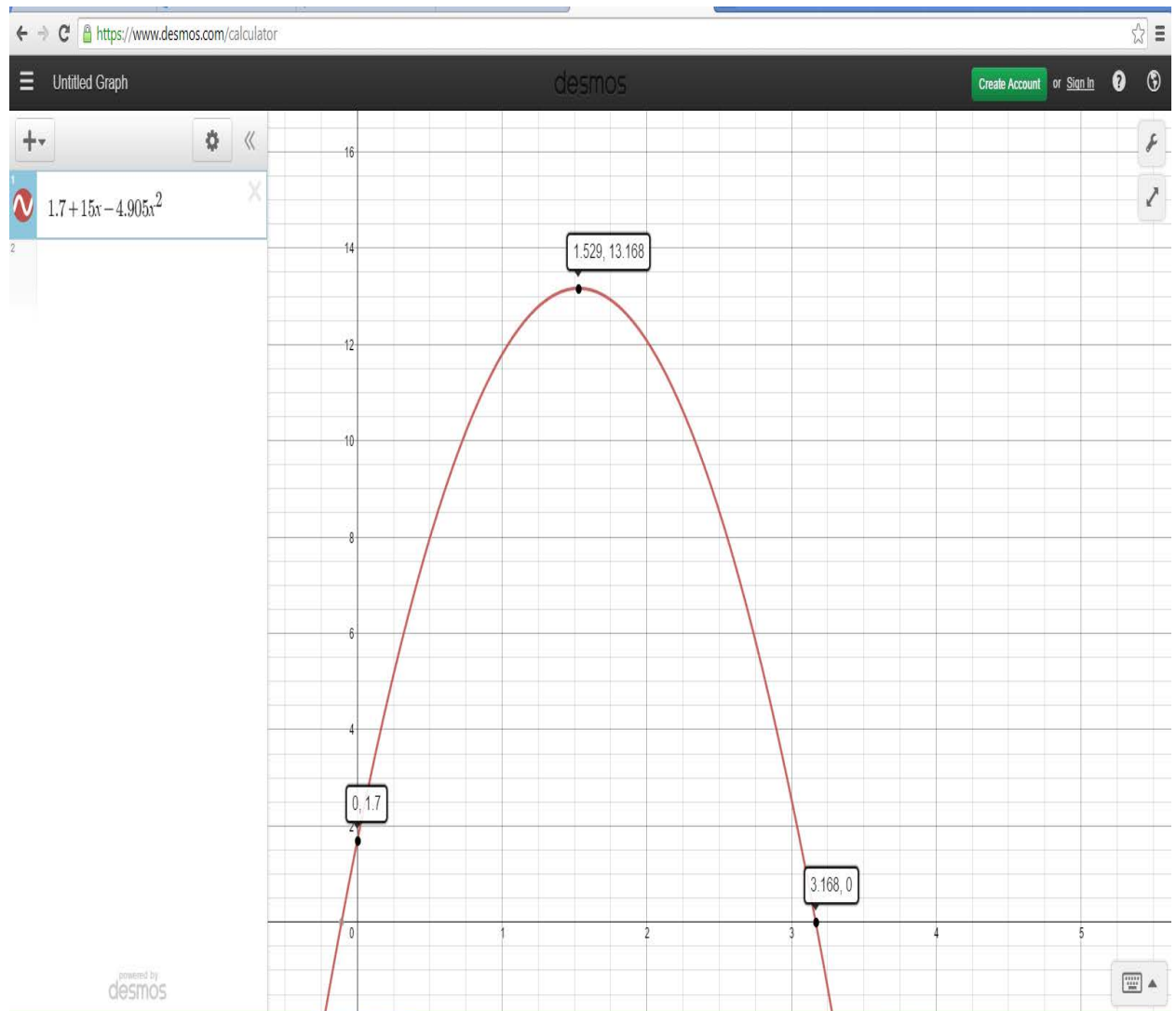
The labeled points are (0,1.7), (1.529,13.168), and (3.168,0). These correspond to the initial height at time 0, the maximum height after 1.529 seconds, and the time that had passed when the ball hit the ground, respectively.

5) <u>3.098 </u>seconds will pass after the football is released from the hands of the quarterback before the receiver could catch the ball at 1.1 meters. The graph of the model showing the path that the football takes after being released from the hands of the quarterback is also given below.

h = 1.7 + 15t – ½ (9.81) t^2

1.1 = 1.7 + 15t – 4.905 t^2

0 = 0.6 + 15t – 4.905 t^2

We decided to graph this quadratic equation (y = 0.6 + 15x – 4.905x^2) in order to find the x-intercept (place where the ball gets in the hands of the receiver). The x-value gives us the amount of time that has passed. We used Desmos Software. Here is the graph:
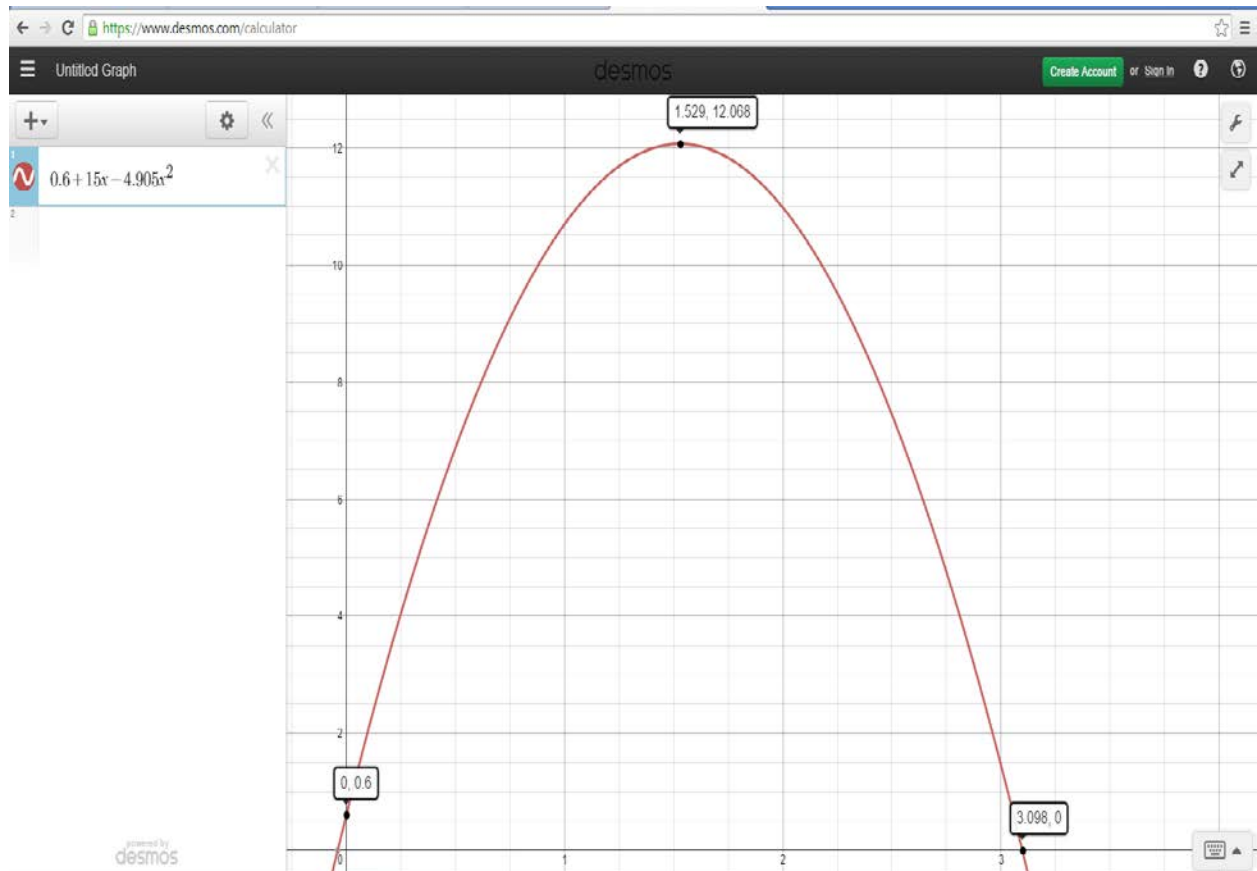
The labeled points are (0,0.6), (1.529,12.068), and (3.098,0). These correspond to the height difference between the quarterback's release position and the receiver's catching position, the maximum height after 1.529 seconds, and the time that passes before the receiver catches the football, respectively.

6) Graph of a model showing the path the football takes after being released from the hands of the quarterback is given in #5.

7) The ball is at the highest point at the maximum point of the parabola. This occurs after 1.529 seconds at 12.068 meters. I know this is true because the maximum point's vertex is at (1.529,12.068). I verified this by using my knowledge of quadratic equations in the form of $ax^2 + bx + c = 0$. The time in which the ball reaches its maximum height and the maximum height can be found by finding $-b/2a$ and then substituting this value back into the equation $y = ax^2 + bx + c$. Here is our work to prove this:

$0 = 0.6 + 15t - 4.905\ t^2$

$-4.905t^2 + 15t + 0.6 = 0$ (a = -4.905, b = 15, c = 0.6)

$-b/2a = -15/2(-4.905)$
$= -15/-9.81$
$= 1.529$ seconds

Substitution (y represents the height):
$y = -4.905t^2 + 15t + 0.6$
$y = -4.905\ (1.529)^2 + 15(1.529) + 0.6$
$y = 12.068$ meters

8) The highest the ball will go is the maximum point, which is the vertex. This information is already mentioned in answer #7.

9) If the receiver runs 30 yards in 5 seconds, when should the quarterback release the ball?

From the information mentioned above, we noticed that it takes the 3.098 seconds from the time the quarterback releases the ball to the time the receiver catches the ball. So, if the receiver runs 30 yards in 5 seconds, we thought it would be a great idea to set up a proportion to find out the distance the receiver could run in 3.098 seconds.

Here is the proportion:

30 yards / 5 seconds = x yards / 3.098 seconds

We cross multiplied and got 30 * 3.098 = 5x, then simplified to get 92.94 = 5x, and then divided both sides by 5 to get 18.588 = x. This means that in 3.098 seconds, the receiver can run 18.588 yards. So, this lets the quarterback know how to time his release. So, if the quarterback recognizes that the receiver has been running for 2 seconds already, prior to his release, he knows that the will have to throw the football further than 18.588 yards. In fact, to find the exact distance, he would have to recognize that the receiver runs at a rate of 6 yards per second, which is 12 yards in 2 seconds. So, the quarterback would have to throw the football for 18.588 + 12 yards or 30.588 yards.

10) At least 2 additional examples of other objects that could also follow a parabolic trajectory: kicking a football and hitting a baseball.